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ECHO EFFECT IN RELATIVISTIC PLASMA

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ABSTRACT. The effects of time echo in relativistic electron plasma are considered. It is shown that repeated excitation of strong damping oscillations in the plasma can cause slight long-wave damping echo oscillations. The limiting electron speed effects the shape of the echo oscillations when relativistic effects are taken into consideration.

Previous papers [1-4] have reviewed echo oscillations in nonrelativistic plasma. Maxwellian distribution was selected as the unperturbed distribution function. The effects of time and space echoes in plasma, occurring as a result of the reaction between two external perturbations, were reviewed. Perturbations with macroscopic magnitudes (with the densities of an electric charge, for example) result in the excitation of natural oscillations in the plasma, the dispersion of which is defined by plasma properties. If external perturbations with microscopic magnitudes are assigned distribution functions, for example, echo oscillations will occur, the dispersion of which will be defined by the nature of the external perturbations. /788*

This paper reviews echo oscillations in collisionless electron plasma, with relativistic effects taken into consideration. Relativistic effects must be taken into consideration when temperatures are high enough, that is, when the thermal energy of the particles can no longer be considered small compared with the potential energy of the particles ($T_e \sim mc^2$, where m is the electron rest mass). However, even in the case of nonrelativistic temperatures, relativistic consideration is necessary when the phenomenon in which we are interested is associated with that part of the pulse-particle distribution for which the particle speed is comparable to the speed of light [5].

1. Basic equations

Ion collisions and movements will be ignored in considering echo oscillations in a relativistic electron plasma. We shall limit ourselves to consideration of longitudinal oscillations. Let us select, as our original set of equations, the

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* Numbers in the margin indicate pagination in the foreign text.

kinetic equation for the electron distribution function and the equation for the self-consistent electric field

$$\frac{\partial f}{\partial t} + \mathrm{v}\frac{\partial f}{\partial \mathrm{r}} + e\mathrm{E}\frac{\partial}{\partial \mathrm{q}}(f_0 + f) = 0, \qquad (1)$$
$$\operatorname{div}\mathrm{E} = 4\pi e \int d\mathrm{q} f + 4\pi\varrho^0,$$

where

ϱ^0 is the density of the external charges;

f is the discrepancy between the distribution function and the function f_0, corresponding to the unperturbed distribution, for which let us use the particle distribution function for a relativistic perfect gas in terms of pulses:

$$f_0(\mathrm{q}) = \frac{n}{4\pi(mc)^3}\frac{e^{-c\frac{\sqrt{m^2c^2+q^2}}{T_e}}}{\left(\frac{T_e}{mc^2}\right)K_2\left(\frac{mc^2}{T_e}\right)}. \qquad (2)$$

Here $K_2(mc^2/T_e)$ is the McDonald function, and n is the electron concentration.

We select the density of the external charges in the form /789

$$\varrho^0(\mathrm{r}, t) = \varrho_1 e^{i\mathrm{k}_1\mathrm{r}}\delta(\omega_0(t-0)) + \varrho_2 e^{i\mathrm{k}_2\mathrm{r}}\delta(\omega_0(t-\tau)), \qquad (3)$$

that is, we take it that the external perturbations are delivered to the plasma at times t = 0, and t = τ (ω_0 is an arbitrary magnitude having the dimensionality of frequency).

Using a Fourier transform in terms of coordinates, and a Laplacian in terms of time, for Eq. (1), we have, in a linear approximation

$$\mathrm{E}^{(1)}_{kp} = -4\pi i\frac{\mathrm{k}\varrho^0_{kp}}{k^2\varepsilon(k, ip)}, \quad f^{(1)}_{kp} = -e\mathrm{E}^{(1)}_{kp}\frac{\frac{\partial f_0}{\partial \mathrm{q}}}{p + i\mathrm{k}\mathrm{v}}, \qquad (4)$$

where

$\varepsilon(k,ip)$ is the longitudinal permittivity for the plasma.

For relativistic plasma ($T_e \lesssim mc^2$) in the case of long-wave oscillations ($ak \ll 1$, where a is the Debye electron radius of screening) the equation

$\varepsilon(k, ip) = 0$ has the solution

$$p = \pm i\tilde{\Omega} - \tilde{\gamma}. \tag{5}$$

In this case, the frequency of plasma oscillations and the decrement can be found by using the formulas [5]

$$\tilde{\Omega}^2 = \frac{4\pi e^2 n c^2}{T_e} K_2^{-1}\left(\frac{mc^2}{T_e}\right) \int_1^{\infty} \frac{dz}{z^2} K_2\left(\frac{mc^2}{T_e} z\right),$$

$$\tilde{\gamma} = \frac{\pi}{4} \frac{\Omega^2 \tilde{\Omega}^2 m}{k^3 c T_e} K_2^{-1}\left(\frac{mc^2}{T_e}\right) e^{-\frac{mc^2}{T_e} \frac{1}{\sqrt{1-\frac{\omega^2}{k^2c^2}}}} \left\{ \frac{1}{1-\frac{\omega^2}{k^2c^2}} + 2\left(\frac{T_e}{mc^2}\right) \times \right.$$

$$\left. \times \frac{1}{\sqrt{1-\frac{\omega^2}{k^2c^2}}} + 2\left(\frac{T_e}{mc^2}\right)^2 \right\} \cdot \begin{cases} 1 & \omega < kc \\ 0 & \omega > kc. \end{cases} \tag{6}$$

For nonrelativistic, and slightly relativistic plasma ($T_e \ll mc^2$), in the case of long-wave oscillations ($ka \ll 1$), the frequency and the decrement equal

$$\operatorname{Im} p \equiv \Omega = \frac{4\pi e^2 n}{m},$$

$$\operatorname{Re} p \equiv \gamma = \sqrt{\frac{\pi}{8}} \frac{\Omega}{(ak)^3} \frac{1}{1-\frac{\omega^2}{k^2c^2}} e^{-\frac{mc^2}{T_e}\left(\frac{1}{\sqrt{1-\frac{\omega^2}{k^2c^2}}} - 1\right)}. \tag{7}$$

Readily seen is the fact that when $\omega/k \ll c$, the expression for γ is converted into an expression for the Landau decrement in a nonrelativistic plasma. We note that in a slightly relativistic plasma, relativistic consideration is necessary for waves, the phase velocity of which can be compared with the speed of light. The basic dependency of field $E^{(1)}$ on time in this case will be exponential, with frequencies and decrements found by using Eqs. (6) and (7), because the contribution of the branch points can be ignored in this case [6]. The contribution /790 of the branch points is significant at temperatures for which $T_e \gtrsim mc^2$ ($T_e \gtrsim 10^9$ °K), according to [6]. In what follows we will consider the electron temperature to be so small compared with electron potential energy that the contribution of the branch points can be ignored.

2. Time echo in plasma

If we take the nonlinear summands in terms of the external perturbations in Eq. (1) we will have

$$\mathbf{E}^{(2)}(\mathbf{r},t)=64\pi^3\frac{e^3Q_1Q_2}{\omega_0^2k_1^2k_2^2k^2}\mathbf{k}e^{i\mathbf{kr}}\left\{(t-\tau)\int d\mathbf{q}\,\frac{(\mathbf{k}_1\mathbf{v})\frac{\partial f_0}{\partial\varepsilon}\mathbf{k}_2\frac{\partial}{\partial\mathbf{q}}(\mathbf{kv})\,e^{-i\mathbf{kv}(t-\tau')}}{\varepsilon(k_1,\mathbf{k}_1\mathbf{v})\,\varepsilon(k_2,\mathbf{k}_2\mathbf{v})\,\varepsilon(k,\mathbf{kv})}+\right.$$
$$+\frac{i\tilde{\Omega}}{2}\int d\mathbf{q}\,\frac{(\mathbf{k}_1\mathbf{v})\frac{\partial f_0}{\partial\varepsilon}\mathbf{k}_2\frac{\partial}{\partial\mathbf{q}}(\mathbf{kv})\,e^{i\mathbf{k}_1\mathbf{v}\tau}e^{-\tilde{\gamma}(t-\tau)}}{\varepsilon(k_1,\mathbf{k}_1\mathbf{v})\,\varepsilon(k_2,\mathbf{k}_2\mathbf{v})}\left[\frac{e^{-i\tilde{\Omega}(t-\tau)}}{(\mathbf{kv}-\tilde{\Omega}+i\tilde{\gamma})^2}-\right.$$
$$\left.\left.-\frac{e^{i\tilde{\Omega}(t-\tau)}}{(\mathbf{kv}+\tilde{\Omega}+i\tilde{\gamma})^2}\right]\right\}, \qquad (8)$$

for field amplitude in the second approximation. Here $\tau' = (k_2v/kv)\tau$. If the integration is made in terms of p' and p, we take it that the condition $\gamma_1\tau, \gamma_2\tau \gg 1$, is satisfied, whereas this condition can be violated for $\gamma\tau$ (when $\tilde{\Omega}/k \to c, \gamma \to 0$). Therefore, the pole from $\varepsilon(k,ip) = 0$ is taken into consideration along with the pole $p = -ikv$ when integration is in terms of p.

Let us consider further that k_2 is antiparallel to k_1, and that $k_2 > k_1$. Now $k = k_2 - k_1$, and $\tau = (k_2/k_2 - k_1)\tau$. It is desirable to change from pulse integration to speed integration in Eq. (8), using the relationship

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dq_x dq_y dq_z \ldots = \int_{-c}^{c} dv_z \int_0^{\sqrt{c^2-v_z^2}} dv_\perp v_\perp \int_0^{2\pi} d\varphi\,|I|\ldots, \qquad (9)$$

Here $|I| = m_0^3/(1 - v^2/c^2)^{5/2}$ is a Jacobian transition. After double integration we have $(T_e < mc^2)$

$$\mathbf{E}^{(2)}(\mathbf{r},t)=64\pi^3e^3\frac{Q_1Q_2}{\omega_0^2k_1k_2k}\mathbf{k}e^{i\mathbf{kr}}\left\{(t-\tau)I_1+\frac{i\tilde{\Omega}}{2}I_2\right\},$$
$$I_1=2\pi T_e m\int_{-c}^{c} dv_z\,\frac{v_z\left(1-\frac{v_z^2}{c^2}\right)^{\frac{1}{2}}\frac{\partial f_0}{\partial\varepsilon}e^{-ikv(t-\tau')}}{\varepsilon(-k_1,-k_1v_z)\,\varepsilon(k_2,k_2v_z)\,\varepsilon(k,kv_z)}, \qquad (10)$$

$$I_2 = 2\pi T_e m \int_{-c}^{c} dv_z \frac{v_z \left(1 - \frac{v_z^2}{c^2}\right)^{\frac{1}{2}} \frac{\partial f_0}{\partial \varepsilon} e^{ik_1 v\tau} e^{-\widetilde{\gamma}(t-\tau)}}{\varepsilon(-k_1, -k_1 v_z)\,\varepsilon(k_2, k_2 v_z)} \times$$
$$\times \left[\frac{e^{-i\widetilde{\Omega}(t-\tau)}}{(kv_z - \widetilde{\Omega} + i\widetilde{\gamma})^2} - \frac{e^{i\widetilde{\Omega}(t-\tau)}}{(kv_z + \widetilde{\Omega} + i\widetilde{\gamma})^2} \right].$$

As distinct from the nonrelativistic case, integration with respect to the longitudinal component of speed is carried out within finite limits. This feature does not permit us to use the methods set forth in [1-4].

Let us use the method of steepest descents ($ks|t - \tau'|$, $kst \gg 1$) [7] to calculate the Eq. (10) integrals. If for example, we substitute the variable $v_z = s/\xi$ (s is the electron thermal speed) in I_1, we have

$$I_1 \sim s^2 \left(\int_{-\frac{s}{c}}^{-\infty} - \int_{\frac{s}{c}}^{\infty} \right) \frac{\left(\xi^2 - \frac{s^2}{c^2}\right)^{1/2} e^{-\frac{2\xi}{\left(\xi^2 - \frac{s^2}{c^2}\right)^{1/2}} - i\frac{ks}{\xi}(t-\tau')} d\xi}{\xi^4 \varepsilon\left(-k_1, -k_1 \frac{s}{\xi}\right) \varepsilon\left(k_2, \frac{k_2 s}{\xi}\right) \varepsilon\left(k, \frac{ks}{\xi}\right)}. \qquad (11)$$

The index of the exponent in Eq. (11) has points of steepest descents $\xi = \pm s/c + (s/2c)^{1/3}/[ks(t - \tau')]^{2/3} \times e^{i(\pi/6)\text{sign}(t - \tau')}$. Let us deform the path of integration in Eq. (11) in the complex plane ξ, and let us at the same time circumvent the pole corresponding to the zeros $\epsilon[-k_1, -k_1(s/\xi)]$, $\epsilon[k_2, k_2(s/\xi)]$, and $\epsilon[k, k(s/\xi)]$. We will take it that the condition $ks|t - \tau'| \gg 1$ has been satisfied. It is easy to show that the portion from the points of steepest descent is immaterial, and that, at the same time, the poles corresponding to the zeros $\epsilon[-k_1, -k_1(s/\xi)]$, when $t < \tau'$, and $\epsilon[k_2, k_2(s/\xi)]$, $\epsilon[k, k(s/\xi)]$, when $t > \tau'$ make the principle contribution to Eq. (11). It can be shown in similar fashion that I_2 in Eq. (10) makes no contribution to the field of echo oscillations.

Let us derive expressions for the field of echo oscillations for a number of concrete cases.

1. Let us select wave numbers k_1, k_2, and k, such that $\tilde{\Omega}/k_1$, $\tilde{\Omega}/k_2$, $\tilde{\Omega}/k \ll c(\gamma_1\tau, \gamma_2\tau, \gamma\tau \gg 1)$. We will take it that the conditions ak_1, ak_2, $ak \ll 1$ have been satisfied. Let us put $k_1 = k$, $k_2 = 2k$. Then, when $t > \tau'$, the contribution from the pole $\epsilon^{-1}[k_2, (k_2 s/\xi)]$ can be ignored, because $\gamma_2 > \gamma$. The expression for the field of echo oscillations in this case will be in the form

$$E^{(2)}(r, t) = -i\frac{32\pi^2}{3}\frac{e\varrho_1\varrho_2\tau}{m\omega_0^2 k^2}\frac{\Omega^3}{\tilde{\Omega}^2} ke^{ikr}\sin\varphi\left(1 - \frac{\tilde{\Omega}^2}{k^2c^2}\right)^{3/2} e^{-\tilde{\gamma}|t-\tau'|}\times \times\cos[\tilde{\Omega}(t-\tau') + \varphi], \tag{12}$$

where phase φ can be found by using the relationship /792

$$\mathrm{tg}\,\varphi = \frac{2k}{k - k_1}\frac{\tilde{\gamma}}{\tilde{\Omega}}.$$

In the case of a slightly relativistic plasma ($T_e \ll mc^2$)

$$E^{(2)}(r, t) = -i\frac{32\pi^2}{3}\frac{e\varrho_1\varrho_2\tau\Omega}{m\omega_0^2 k^2} ke^{ikr}\sin\varphi\left(1 - \frac{\Omega^2}{k^2c^2}\right)^{3/2} e^{-\gamma|t-\tau'|}\cos[\Omega(t-\tau') + \varphi]. \tag{13}$$

Since $k_1 = k$, the shape of the echo signal is symmetrical with respect to time. We note that when $\tilde{\Omega}/k \ll c$, Eq. (12) coincides with the expression for the field of Langmuir echo oscillations for nonrelativistic plasma obtained earlier in [1].

2. Let us select wave numbers for the external perturbations k_1 and k_2 such that $\tilde{\Omega}/k_1$, $\tilde{\Omega}/k_2 \ll c$, and that their difference $k = k_2 - k_1$ is such that the condition $\tilde{\Omega}/k \lesssim c$, is satisfied. Moreover, we will take it that the conditions ak_1, $ak_2 \gg 1$, $ak \ll 1$ are satisfied. Then the poles corresponding to the solutions $\epsilon(-k_1, ip) = 0$, and $\epsilon(k_2, ip) = 0$ will describe the heavily damped oscillations, and the pole corresponding to the equation $\epsilon(k, ip) = 0$ will describe the slightly damped oscillations. Readily seen is the fact that when $t < \tau'$, the field of echo oscillation will increase sharply, and then will decrease slowly when $t > \tau'$.

The expression for the field of echo oscillations when $t > \tau'$ is in the form

$$E^{(2)}(r,t) = -i32\pi^2 \frac{e\varrho_1\varrho_2\Omega^3\gamma}{m\omega_0^2 k_1 k_2 \tilde{\Omega}^2}(t-\tau)\left(1-\frac{k\tilde{\Omega}^2}{k^2c^2}\right)^{3/2} \mathbf{k} e^{ikr} e^{-\tilde{\gamma}(t-\tau')}\cos(\Omega(t-\tau')). \tag{14}$$

When $\Omega/k \to c$ $\gamma \to 0$, the echo oscillations that do occur decrease slowly with time.

3. If the wave numbers of the external perturbations, k_1 and k_2, are selected such that $\tilde{\Omega}/k_1$, $\tilde{\Omega}/k_2 \lesssim c$, and if $\tilde{\Omega}/k \gg c$, then the pole corresponding to the solution of equation $\epsilon(k, kv_2) = 0$ lies outside the contour of integration. In this case the expression for the field of echo oscillations is in the form

$$E^{(2)}(r,t) = i32\pi^2 \frac{e\varrho_1\varrho_2(t-\tau)}{m\omega_0^2 k^2}\frac{\Omega^3}{\tilde{\Omega}^2}\mathbf{k}e^{ikr} \times$$
$$\times \begin{cases} \gamma_1\left(1-\dfrac{\tilde{\Omega}^2}{k_1^2c^2}\right)^{3/2} e^{\frac{k}{k_1}\gamma_1(t-\tau')}\cos\left(\dfrac{k}{k_1}\tilde{\Omega}(t-\tau')\right), & t<\tau' \\ \gamma_2\left(1-\dfrac{\tilde{\Omega}^2}{k_2^2c^2}\right)^{3/2} e^{-\frac{k}{k_2}(t-\tau')\gamma_2}\cos\left(\dfrac{k}{k_2}\tilde{\Omega}(t-\tau')\right), & t>\tau'. \end{cases} \tag{15}$$

Frequency and decrement can be found by using Eq. (6).

Thus, electrons with a limiting speed in a relativistic plasma have a significant effect on the shape of the echo oscillations. /793

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